Transition Curves: A Comparison of Three Methods

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TRANSITION CURVES

A COMPARISON OF THREE METHODS

HOWARD'S TRANSITION CURVE, — SEARLE'S RAILROAD SPIRAL — TALBOT'S TRANSITION SPIRAL

PART I. OUTLINE AND COMPARISON OF THE THEORY OF EACH, WITH SUGGESTONS.

PART II THE DEVELOPMENT OF THE FUNDAMENTAL TABLES OF EACH AND THEIR APPLICATION TO CERTAIN PROBLEMS.

PART III THE METHODS IN GENERAL

Thesis by John H. Fletcher
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Equations for Reference

(1) $g = \frac{m}{3} = \frac{x}{4}$

(2) $x = 4g = \frac{4}{3}m = \frac{D^\circ n^2}{34380}$

(3) $x : x_1 :: n^3 : n_1^3$

(4) $D^\circ : D_1^\circ :: n : n_1$

(5) $\Delta = \frac{D^\circ n}{200}$

(6) $\Delta : \Delta_1 : n^2 : n_1^2$

(7) $\sin \phi = \frac{4g}{m}$

(8) $2 \sin \frac{1}{2} \Delta = \frac{6g}{n}$

(9) $\phi = \frac{\Delta}{3}$

(10) $x = ay^3$

(11) $\tan\Delta = 3ay^2$

(12) $R = \frac{5730}{D^\circ} = \frac{(1+9a^2y^4)^{3/2}}{6ay}$

(13) $\tan \phi = ay^2$

(14) $n = y + \frac{9}{10} a^2 y^3 - \frac{9}{8} a^4 y^9$

$(1)^2$ $wL = \frac{wn}{100}$

$(2)^2$ $\Delta = \frac{wL^2}{114.6}$

$(3)^2$ $\Delta = \frac{wL^2}{2}$

$(4)^2$ $X = \frac{1070.5}{(w)^{1/2}} \left\{ \frac{1}{3} \Delta^{3/2} - \frac{\Delta^{7/2}}{42} + \frac{\Delta^{11/2}}{1320} - \text{etc} \right\}$

$(5)^2$ $y = \frac{1070.5}{(w)^{1/2}} \left\{ \Delta^{1/2} - \frac{\Delta^{5/2}}{10} + \frac{\Delta^{9/2}}{216} - \text{etc} \right\}$

$(6)^2$ $X = .291 wL^3 - .0000158 w^3 L^7$

$(7)^2$ $y = 100L - .000762 D^2 L^3$

$(8)^2$ $\phi = \frac{1}{3} \Delta$

The following notation will be used. See. Fig I.

S = P.S. or Point of spiral or Beginning of Spiral

L = P.CC the point where spiral compounds with the circular curve.

P.C. = beginning of offsetted circular curve in Fig I it is the point D.

R = radius of curvature of the Spiral at any point

D = degree-of-curve of the spiral at any point

n = length of spiral

Δ = angle showing the change of direction at the spiral at any point on the spiral

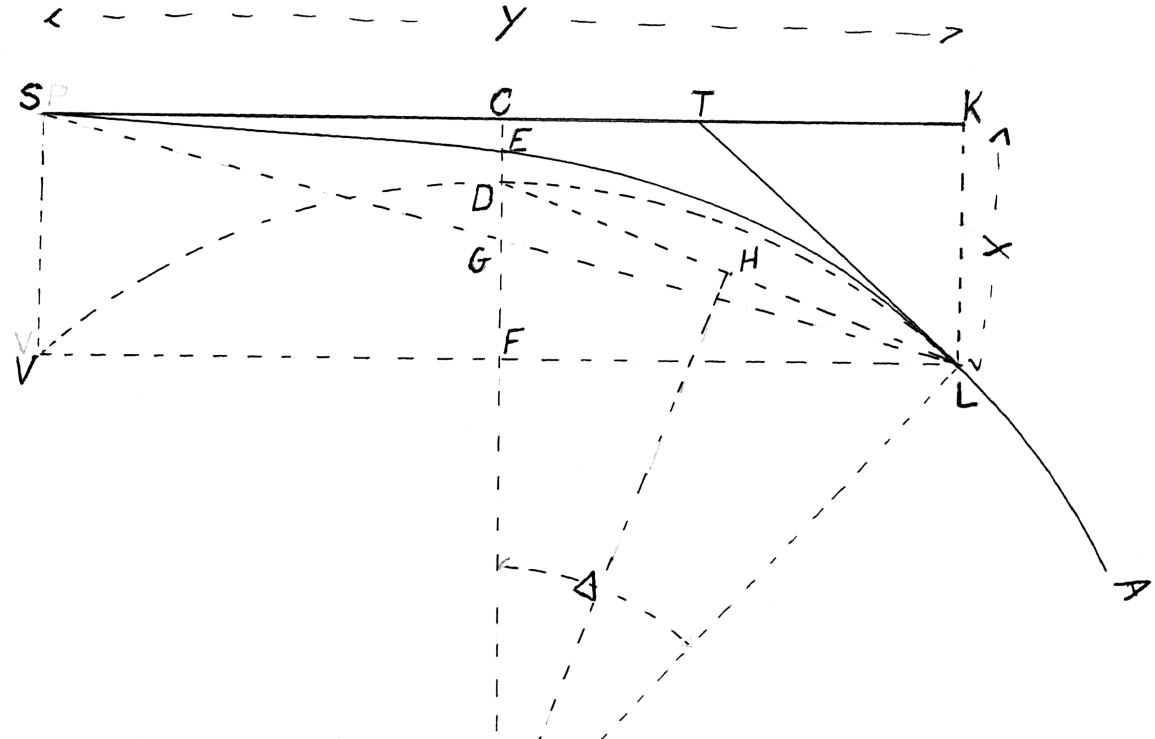
ϕ = deflection angle at the P.S. from the tangent SK to any point on the spiral,

y = abscissa of any point on the spiral, referred to the P.S. as the origin and the tangent SK as the axis of y.

X = ordinate of the same point, measured at right angles to the above axis.

q = CD m = DF

Fig I



The Theory of The Transition Curve as Developed by Mr. Howard.

General Proposition

If a tangent SK. and a circular arc D.A. are to be connected by a cubic parabola SEL of central angle Δ, the two curves to have a common tangent LT and the same rate of curvature at their junction L, the adjusted gap C.D between tangent and circular curve will be one third of the middle ordinate FD of the circular arc of 2Δ and one fourth of the terminal ordinate LK of the parabola.

The Proof

In a cubic parabola SEL $\quad CE : KL :: \overline{SC}^3 : \overline{SK}^3$

By construction $\quad SC = \frac{1}{2} SK$

$\therefore\ CE : KL :: \frac{1}{8} : 1$ and $CE = \dfrac{KL}{8}$

By construction $\quad CD = \dfrac{FD}{3} = \dfrac{FC}{4} = \dfrac{LK}{4}$

Hence $C.D = 2CE$ and, consequently the cubic parabola SEL will pass midway between C and D.

Denoting the middle ordinate F.D of 2Δ by m and the gap CD by g and the end ordinate LK by x then by substitution in last equation $g = \frac{m}{3} = \frac{x}{4}$ . . . (1)
Which was to be prooved.

Also at any other P.T. on S.E.L. $g_1 = \frac{m_1}{3} = \frac{x_1}{4}$

Denoting the radius of the circular arc LD by R the parabola length SL by n and taking $LD = \frac{n}{2}$; from Similar triangles LDF and LOH

$$R : \frac{n}{4} :: \frac{n}{2} : m \quad \text{and} \quad m = \frac{n^2}{8R}$$

Substituting $\frac{5730}{D^\circ}$ for R and solving

$$x = 4g = \frac{4}{3} m = \frac{D^\circ n^2}{34380} \quad \ldots\ldots (2)$$

Also at any other Pt. on S.E.L $x_1 = 4g_1 = \frac{4}{3} m = \frac{D_1^\circ n_1^2}{34380}$

Whence $\quad x_1 : x :: D^\circ n^2 : D_1^\circ n_1^2$

But x and x_1 being ordinates to a Cubic Parabola approximately $x : x_1 :: n^3 : n_1^3$ . . . . . . (3)

Therefore $\quad D^\circ : D_1^\circ :: n : n_1$ . . . . . . (4)

The central angles Δ and Δ_1 turned in the parabola from S to L and L_1 respectively, are measured by the circular arcs of length $\frac{n}{2}$ and $\frac{n_1}{2}$ and of degrees of curve D° and D_1° per 100 Ft.

Therefore $\Delta = \frac{D^\circ n}{200} \quad \Delta_1 = \frac{D_1^\circ n_1}{200}$ . . . . . . (5)

Substituting (5) in (4) and $\Delta : \Delta_1 :: n^2 : n_1^2$ . . . . . . (6)

From (3) (4) and (6) we have the three proportions

I The ordinates of the Transition Curve are as the cubes of the spiral lengths.

II The central angles of the Transition Curve are as the squares of the Spiral Lengths.

III The degrees of curves of the Tangent Curves are as the Spiral Lengths.

From triangle G.F.L

$$\sin GLF = \sin GSK = \sin \phi = \frac{FG}{LG}$$

But approximately $LG = \frac{n}{2}$ and $FG = 2g$

Substituting in last equation and $\sin \phi = \frac{4g}{m}$ ..... (7)

From triangle LDF

$$\sin \tfrac{1}{2}\Delta = \frac{DF}{DL}$$

But approximately $DL = \frac{n}{2}$ and $DF = 3g$

Substituting and multiplying both sides of the above equation by 2 and

$$2 \sin \tfrac{1}{2}\Delta = \frac{6g}{n} \qquad \text{.............. (8)}$$

For small values of Δ and ϕ we have from (7) and (8) $\phi : \Delta :: 1 : 3$

$$\therefore \phi = \frac{\Delta}{3} \qquad \text{.................... (9)}$$

It seems as if Mr Howard has gone to considerable extra and tedious work to derive the above formula and proportions. All of them can be derived from the cubic parabola equation as follows.

$$x = ay^3 \text{ and } x_1 = ay_1^3 \qquad \ldots\ldots\ldots\ldots (10)$$

$$\therefore x : x_1 :: y^3 : y_1^3$$

Approximately $y = n$ the length of parabola

Hence equation (3) $x : x_1 :: n^3 : n_1^3$

$$\tan\Delta = \frac{dx}{dy} = 3ay^2 \qquad \ldots\ldots\ldots\ldots (11)$$

For small Δs $\tan\Delta = \Delta$

$$\therefore \Delta = 3ay^2 \text{ and } \Delta_1 = 3ay_1^2$$

Hence equation (6) $\Delta : \Delta_1 :: y^2 : y_1^2$

$$R = \frac{5730}{D^\circ} = \frac{ds}{d\Delta} = \frac{\left[1+\left(\frac{dx}{dy}\right)^2\right]^{3/2}}{\frac{d^2x}{dy^2}}$$

But $\frac{dx}{dy} = 3ay^2$ and $\frac{d^2x}{dy^2} = 2ay$ and substituting

in the above equation

$$R = \frac{5730}{D^\circ} = \frac{(1+9a^2y^4)^{3/2}}{6ay} \qquad \ldots\ldots\ldots\ldots (12)$$

Hence $D : D_1 :: \frac{(1+9a^2y^4)^{3/2}}{(1+9a^2y_1^4)^{3/2}} \times \frac{y}{y_1}$

For Mr Howards limit of accuracy the expression $\frac{(1+9a^2y^4)^{3/2}}{(1+9a^2y_1^4)^{3/2}} = 1$ Hence equation (4) $D^\circ : D_1^\circ :: y : y_1 :: n : n_1$

From equation (11) $\Delta = 3ay^2 = \tan\Delta$

The $\tan\phi = \frac{x}{y} = \frac{ay^3}{y} = ay^2$ . . . . (13)

Hence $\tan\phi : \tan\Delta :: 1:3$ for small values of Δ and ϕ we have equation (9) $\phi = \frac{\Delta}{3}$

Mr Howard would probably answer the above criticsm by reference to Paragraph 2 of his Prefatory. viz — "In the investigation of the principles upon which the rules are based, it will be seen that with data consisting in great part of familiar approximations used in circular-curve location, and with no mathematics beyond a little algebra and trigonometry, practically exact results are reached in regard to laws of the Transition Curve and its relations to circular curves."

In answer to this I will say that to prove his General Proposition, Mr Howard has assumed properties of the Cubic Parabola and consequently it would have been as easy and clear, to the engineer not familiar with calculus, to have started with equations (10) (11) and (12)

In order to make my cubic parabola method complete an equation for the value of $\underline{n}$ in term's of y can be developed

By calculus $n = \int_0^y \left[1+\left(\frac{dx}{dy}\right)^2\right]^{1/2} dy$ But $\frac{dx}{dy} = 3ay^2$

$$\therefore n = \int_0^y (1+9a^2y^4)^{1/2} dy$$

But $(1+9a^2y^4)^{1/2} = 1 + \frac{9a^2y^4}{2} - \frac{8a^4y^8}{8} + \ldots\ldots\ldots\ldots$

Hence $n = \int_0^y dy + \int \frac{9a^2y^4 dx}{2} - \frac{8a^4y^8 dx}{8} + \ldots\ldots\ldots$

$$\therefore n = x + \tfrac{9}{10} a^2y^3 - \tfrac{9}{8} a^4y^9 \ldots\ldots\ldots\ldots\ldots\ldots\ldots - (14)$$

For practical use all terms beyond the second can be dropped.

For furthur discussion of the Cubic Parabola Theory see Howard's Transition-Curve Field Book.

Talbot's Theory of the Transition Spiral

By definition The Transition Spiral is a curve whose degree-of-curve increases directly as the distance along the curve from the point of spiral.

Let w = rate of change of the degree-of-curve of the spiral per 100 Ft. of length. It is the degree-of-curve of spiral at 100 Ft. from the P.S. or point of spiral – Let L = no of stations and n = length of curve.

Whence D = degree curve $= wL = \frac{wn}{100}$ . . . . (1)[2]

From calculus the radius of curvature

$R = \frac{dn}{d\Delta}$ where Δ is the central angle of the spiral as shown in figure I.

But $R = \frac{5730}{D} = \frac{573000}{wn}$

$\therefore d\Delta = \frac{wn\,dn}{573000}$ integrating, $\Delta = \frac{wn^2}{1146000} = \frac{wL^2}{114.6}$ . . . . (2)[2]

Changing Δ from circular measure to degrees and

$\Delta = \frac{1}{2} wL^2$ . . . . (3)[3]

To find the co-ordinates x and y, of any point on the spiral, we have by calculus $dx = dn \sin \Delta$ and $dy = dn \cos \Delta$. Expanding the sin and cosine into an infinite series, substituting for ~~$d\Delta$~~ dn its value in terms of $d\Delta$, and integrating we have

$X = \frac{1070.5}{(w)^{1/2}} \left\{ \frac{1}{3}\Delta^{3/2} - \frac{\Delta^{7/2}}{42} + \frac{\Delta^{11/2}}{1320} - \text{etc} \right\}$ . . . . (4)[2]

And $y = \frac{1070.5}{(W)^{1/2}} \left\{ \Delta^{1/2} - \frac{\Delta^{5/2}}{10} + \frac{\Delta^{9/2}}{216} . \text{ etc} \right\} \ldots\ldots\ldots\ldots (5)^2$

Changing the angle Δ from circular measure to degrees, substituting for Δ, and dropping the small terms

$$x = .291\, wL^3 - .00000158\, w^3 L^7 \ldots\ldots\ldots\ldots (6)^2$$

$$y = 100L - .000762\, D^2 L^3 \ldots\ldots\ldots\ldots (7)^2$$

To find the deflection angle ϕ for any point on the spiral, as K.S L for the point L

Divide equation $(4)^2$ by $(5)^2$ and

$$\tan\phi = \tfrac{1}{3}\Delta + \frac{\Delta^3}{105} + \frac{26\Delta^5}{155925} \text{ etc}$$

But from tangent series

$$\tan \tfrac{1}{3}\Delta = \tfrac{1}{3}\Delta + \frac{\Delta^3}{81} + \frac{2\Delta^5}{3645}$$

The error of calling the two equations equal is less than a' for $\Delta = 25°$ and decreases rapidly below this

There fore $\phi = \tfrac{1}{3}\Delta \ldots\ldots\ldots\ldots (8)^3$

From $(1)^2$ $\qquad D : D_1 :: n : n_1$

" $(2)^2$ $\qquad \Delta : \Delta_1 :: n^2 : n_1^2$

" $(6)^2$ for small values of Δ $\qquad x : x_1 :: n^3 : n_1^3$

And $(8)^3$ is the same as (9)

Hence the above theory developes the same proportions as were developed by the cubic parabola method. For small values of Δ it shows that

Howards Transition Curve is practically the same as Talbots Transition Spiral

Mr. Talbots theory is pretty, well developed and easily understood.

The Theory of Searles Railroad Spiral

"The Railroad Spiral is a compound curve closely resembling the cubic parabola. It is constructed upon a series of chord'z of equal length, and the curve is compounded at the end of each chord. The chords subtend circular arcs, and the degree-of-curve of the 1st arc is made the common difference for the degrees of curve of the 1st arc be 0°10' that of the second will be 0°20', of the 3rd 0°30' etc."

From the above it is evident that the Theory of The Railroad Spiral is the same as the theory of Compound curves. Mr Searles treats the subject as such and developes his formula and tables accordingly.

Asumming of the three methods show the following.

I The Railroad Spiral Theory is the same as that of compound curves.

II For small values of Δ Howard's Transition Curve is practically the same as Talbot's Transition Spiral

III The development of Talbot's Theory is more scientific and easier to follow than that of Howard's.

IV Howard's method could have been easier and more scientifically developed had he used the Cubic Parabola Equation as suggested by the writer.

PART II

The Development of Howard's Fundamental Table

By Fundamental Table is ment the one used in the selecting of a Spiral. All three of the treatises on this Subject contain other tables for different purposes but I have treated them only in a general way in PART III.

In making out his Table (See Table I) Mr Howard has let $\Delta = 06'$ and c = cord length = 100 and $N = 1$. This limits us to one particular Cubic Parabola. It means that Δ increases from $0°$ to $06'$ in 100 Ft, from $0°$ to $24'$ in 200 Ft. etc. See equation (6). It gives us a certain value of a in the Cubic Parabola equation $x = ay^3$

From (11) $\Delta = 3ay^2$

For small values of Δ $y = n$ But for $\Delta = 06'$

$n = c \times N = 100$

Substituting in (11) and $.1° = 3a \times 10000$

$\therefore a = .00003333$ and hence the Parabola equation becomes $x = .00003333\,y^3$ for this particular equation.

The ordinates x and y in Table I were figured from $x = \frac{4}{3} R \text{ versin}\,\Delta$ and $y = \frac{n}{2} + R \sin\Delta$ after calculating R by the use of equation (5) The or-

TABLE I

N	S	1	2	3	4	5	6	7	8	9	10	11	12
X.	0	.0582	.4654	1.571	3.723	7.271	12.563	19.946	29.761	42.352	58.053	77.191	100.083.
Y.	0	100	200	299.99	399.97	499.91	599.77	699.49	799.01	898.22	996.97	1095.13	1192.47.
F.	0	.25	.25	.25	.25	.25	.25	.25004	.25015	.25028	.25047	.25072	.25105.
Q.	0	.0029	.0465	.2356	.7445	1.818	3.769	6.982	11.912	19.080	29.081	42.577	60.302.
Z.	0	20	80	180	320	500	720	980	1280	1620	2000	2420	2880.
C.	0	10	40	90	160	260.5	359.8	489.4	638.7	807.3	994.9	1201.0	1424.9.
Δ φ.	0	0.06′	0 24′	0 54′	1°36′	2°30′	3°36′	4°54′	6°24′	8°06′	10°00′	12°06′	14°24′.
& Δ.	S	0 02′	0 08′	0 18′	0 32′	0 50′	1°12′	1°38′	2°08′	2°42′	3°20′	4°02′	4°48′.

dinates were also recalculated and checked by summing the products of each chord into the sine and cosine respectively of its angle of inclination to the tangent SK.

The ordinates x and y thus obtained are for the chord stations of a Transition curve of 100 Ft. chords; But for any other chord length the corresponding ordinates x and y are directly proportional.

The other quantities in Column n are

$F = \frac{g}{x}$ $Q = gD$ $Z = Dn$ C is not need for the choice of a spiral. Δ and φ have been explained before

The quantities in tabl I are for the particular cubic parabola $x = .00000333\,y^3$ but by choosing any other chord length we can determine any number of other spirals by the same prin cipal that we can determine the properties of a $5°$ curve if we know those of a $1°$ curve for the same central angle. But owing to approximations made in developing the theory the Table is not accurate for values of Δ greater than $15°$. For close work Δ ought not to be larger than $10°$. Hence the table is limited and cannot be used in many cases for high degree curves. I

If Mr Howard had used the cubic parabola equation he could have made an accurate table for all values of Δ that are ever needed.

Table I is only a part of what is given in Howards Field Book. This is touch upon in PART III.

Talbots Tables.

By giving w a certain value we determine one certain Transition Spiral according to Mr Talbots method. For example Let $w=1$

Substituting in equation $(3)^2$ and $\Delta = \frac{L^2}{2} = \frac{m^2}{20000}$

It only remains now to give values to Δ, m, x or y to determine points on the spiral. By giving values of 10 - 20, 30 and 40 etc to m the other quantities can be calculated and tabulated as Mr Talbot has done in Table II

TABLE II

LENGTH	D	Δ	α	o	Y	xCOR	tCOR.
10	0.1°	0°00.3'	0°00.1'	.000	0.000	0.0000	0.0000
20	0.2°	01.2	00.4	.001	.002		
30	0.3	02.7	00.9	.002	.008		
40	0.4	04.8	01.6	.005	.019		
50	0.5	07.5	02.5	.009	.036		
60	0.6	0 10.8'	03.6	.016	.063	.0001	
70	0.7	14.7'	04.9	.025	.100	.0001	
80	0.8	19.2	06.4	.037	.149	.0002	
90	0.9	24.3	08.1	.053	.212	.0004	.0001
100	1.0°	30.	10.0	.073	.291	.0008	.0001

In Mr Talbots Book Table II is extended to include a length of 700'.

Mr Talbot has given different values to w and has calculated values for eight different Spirals and tabulated the results. with these eight tables a sutible spiral can be found for most any case which may come up.

For explanation of o, xcor and tcor in Table II see Talbot's Transition Spiral.

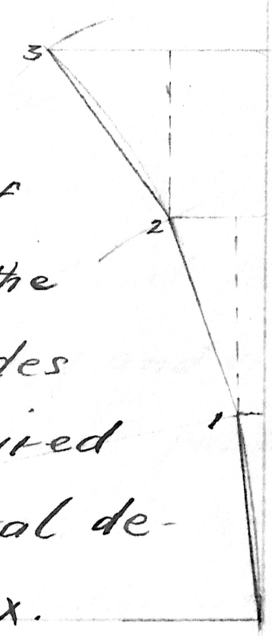
Searles Tables

For the purpose of calculation Searles begins with a compound curve such that the 1st 100 ft chord has a central angle of 10′ the second 20′ the 3rd 30′ etc.

The angles which the chords make with the tangent SY are

1st chord $\frac{1}{2} \times 10' = 05'$
2nd " $10' + \frac{1}{2}\, 20' = 20'$
3rd " $10' + 20' + \frac{1}{2}\, 30 = 45'$
&c. &c &c

FIG. 2.



"If we consider the tangent as a meridian, the Latitude of a chord will be the product of the chord by the sine of its inclination to the tangent. Aa summation of the several Latitudes for a series of chords will give us the required values of y and a summation of the several departures will give us the required values of x.

By the aid of a table of sines and cosines, we may therefore readily prepare Table III

We have assumed the spiral to be constructed upon chords of 100 ft, but it is evident that such a spiral would be entirely to long for practical use. It would be 1700 ft. long before reaching a 3° curve. ~~The values of x and y and of the radii of the arcs at correspond~~

Table III

Point N	Degree of Curve D	Spiral angle Δ	Inclination of chord to axis of Y	Latitude of each chord 100 × cos. Incl.	Sum of the Latitudes Y	Departure of each chord 100 × sin. Incl	Sum of the Departures X
0	0°00′	0°00′	0° 00′	00.000 0	00.0000	0.00000	0.00000
1	10′	10′	05′	99.9998	99.9998	0.14544	0.14544
2	20′	30′	20′	99.9983	199.9982	.581773	.727217
3	30′	1° 00	45′	99.9914	299.9896	1.30895	2.03617
4	40′	1° 40′	1°20′	99.9729	399.9625	2.32689	4.363072
5	50′	2°30′	2°05′	99.9339	499.8964	3.63530	7.99837
6	1°	3°30′	3°00′	99.9629	599.7594	5.23359	13.23196
&c	&c	&c	&c	&c.	&c	&c	&c.

By shortening the chord length we merely construct the spiral on a smaller scale. The values of X and y and of the radii of the arcs at corresponding points are proportional to the chord lengths, and the degrees of curve for corresponding chords are inversely proportional to the same. In this way Mr Searles has constructed 41 tables similar to Table III, with values of chord lengths from 10 Ft to 50 Ft varying by a single foot. This set of Tables will answer, for most purposes.

In order to show the practical results in the application of the three methods I have limited myself to certain conditions and have selected a the following set of curves to be eased off.

Spirals are very seldom run in at the time a line is located. Not until traffic increases and there is a demand for fast trains does the Chief Engineer bother ~~me~~ himself about easement. But by this time the roadbed is in good shape, and consequently we must put in that spiral which will cause the least change of roadbed

In order to have something definite to work with I selected one hundred curves each of a 1°, 2°, 3°, 4° and a 5° curve from the Eastern Division of the Santa Fe System and tabulated the following.

TABLE IV

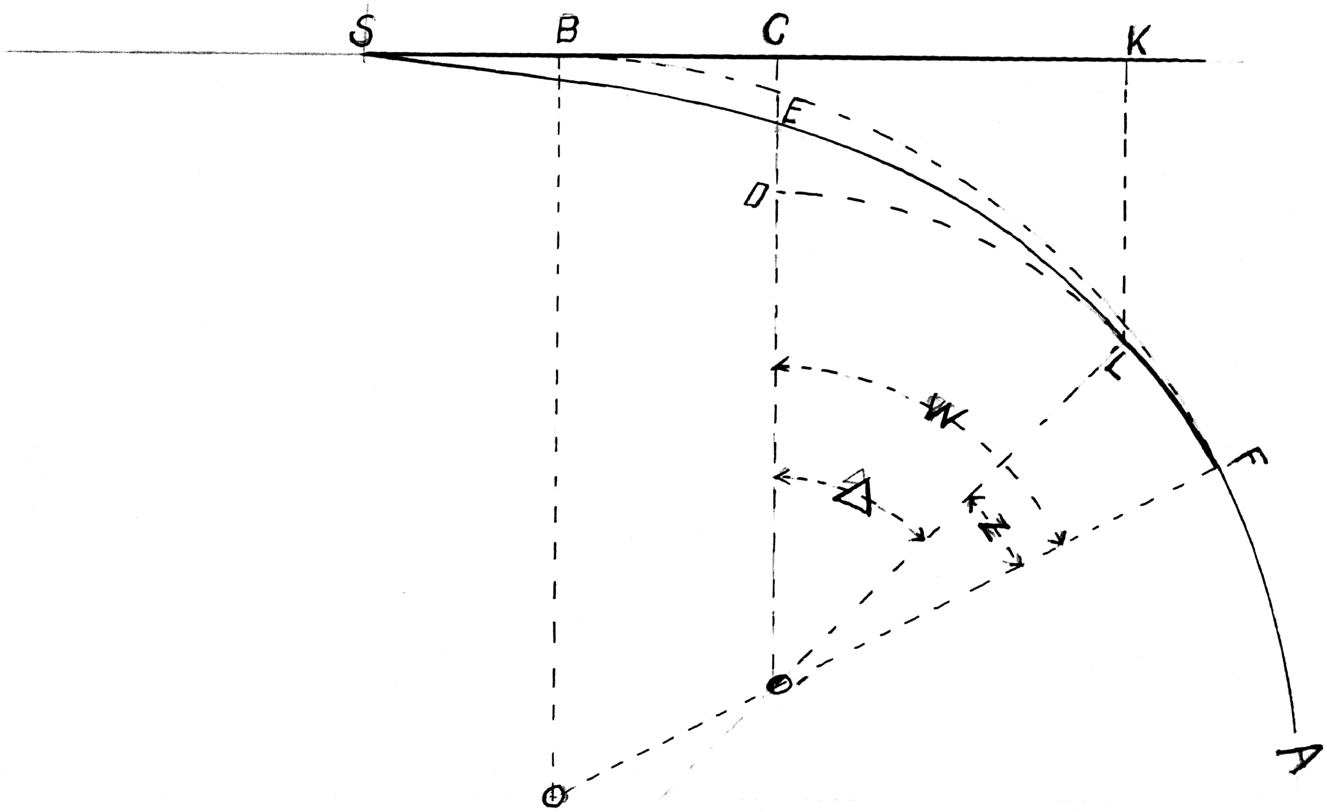
Degree of Curve	Curve with smallest central Angle	Curve with average central Angle	Curve with greatest central Angle
1°	1° 45'	7° 00'	35° 30'
2°	4° 51'	15° 00'	53° 00'
3°	13° 32'	24° 30'	61° 59'
4°	7° 08'	24° 15'	63° 30'
5°	6° 25'	23° 30'	50° 29'

The smallest 1° curve contained 1° 45' of Cental Angle
" average " " " 7° 00' " " "
" largest " " " 35° 00' " " "
&c &c &c &c

In order to make the least change in roadbed I have used this problem

Given, a simple curve joining two tangents, To compound the curve near each end, with an arc and spiral joining the tangent, without disturbing the middle portion of the curve. Fig This problem is found on page 34 of Searle's Book, on page 40 of Howard's Book, on page 37 of Talbot's Book.

Fig. 3.



BFA is the origonal curve as it is on the ground
SEL " " spiral curve
LF is the arc of a circle ~~with~~ of a little greater degree than the origonal curve. It is compounded

TABLE V

By Howard's Tables

D	D^c	Δ	W	Z	SEL	LF	SB	BF	SELF	SBF
1°	1° 12′	0° 54′	1° 17′	0° 23′	150.00	31.9	53.61	128.33	181.9	181.94
2°	2° 06′	2° 30′	6° 28′	3° 58′	238.1	188.88	103.44	323.33	426.98	426.77
3°	3° 07½	3° 36′	10° 12′	6° 36′	230.4	211.2	101.66	340.00	441.66	441.66
4°	4° 12′	4° 54′	12° 40′	7° 46′	233.33	184.91	101.67	316.66	418.24	418.33
5°	5° 30′	6° 24′	11° 42′	5° 18′	232.73	96.4	95.13	234.00	329.12	329.13

By Talbot's Tables

1°	1° 12′	0° 43.2′	0° 55′	0° 12′	120.0	16.39	44.72	91.666	136.39	136.38
2°	2° 06′	2° 12′	5° 42′	3° 30′	210.0	166.43	91.45	286.00	376.43	376.46
3°	3° 07½	3° 54′	11° 04′	5° 10′	250.0	229.13	110.35	368.88	479.13	479.23
4°	4° 12′	4° 25′	11° 25′	7° 00′	210.0	166.8	91.5	285.41	376.9	376.9
5°	5° 30′	6° 03′	11° 03′	5° 00	220.0	91.0	90.04	221.00	311.04	311.04

By Searles Tables

1°	1° 12′	1° 00′	1° 00′	0° 00′	126.0	0.000	26.016	100.00	126.00	126.01
2°	2° 06′	2° 30′	5° 45′	3° 15′	206.00	154.76	72.27	287.50	359.76	359.77
3°	3° 07½	4° 40′	11° 09′	6° 29′	259.00	207.46	94.86	371.66	466.46	466.52
4°	4° 12′	4° 40′	10° 37′	5° 57′	196.00	141.66	72.28	265.04	337.66	337.69
5°	5° 30′	6° 00	10° 22′	4° 22′	200.00	79.39	72.10	207.33	279.39	279.43

with the spiral at L and with the origional curve at F.

W° = the amount of old curve tobe taken out.

Δ = central angle of spiral

Z = Amt. of the curve LF

Quantities in TABLE V. not mentioned above are D = degree of origional curve

D^c = " " curve LF

SEL = length of spiral

LF = " " arc

SB = distance of old P.C. to point of spiral

BF = length of old curve taken out

SELF = Total amt. of new line

SBF = " " " Old " taken out.

TABLE V. is made to fit the average central angles as shown in TABLE IV Hence the W's ought not to be larger than ½ their corresponding central angles as shown in Table IV.

The quantities in Table V that means the most are the values of W (which determine the Amt. of old curve tobe thrown out.)

Searles Method seem's to give smaller values for W for the selected set of curves. This is to be desired here. Talbots Method comes next

Had a different set of curves been selected Howard's method might have proaved the best.

Practically there will, on an average, be very little difference in the results obtained from Talbots Tables and from those of Searles. But the Δ in Howard's Table is limited to $14°24'$ and consequently cannot be used in many cases of curves with high degree.

PART III

Without going into furthar detail I would advise the use of Searles Spirals in locating. His book contains a completer set of problems and diflec-tions Tables which are time savers.

For more deliberate and careful work I would recommend Talbots method. His is the real true spiral. By this method a right angle can be turned at any point on the spiral. This can not be done by the Searles method. With a few diflection tables added to Mr Talbots Book it could be made the most practical of all.

Owing to the approximations made in develop ing Howards Theory there is too short a limit to its accuracy. It cannot compare with the other two methods. But by the use of the Cubic Para-bola equation, a method as scientific and as prac tical as Talbots could be developed.

It is the intention of the write to further develop the cubic parabola method as soon as time affords.

Respectfully submitted,

J. H. Fletcher,